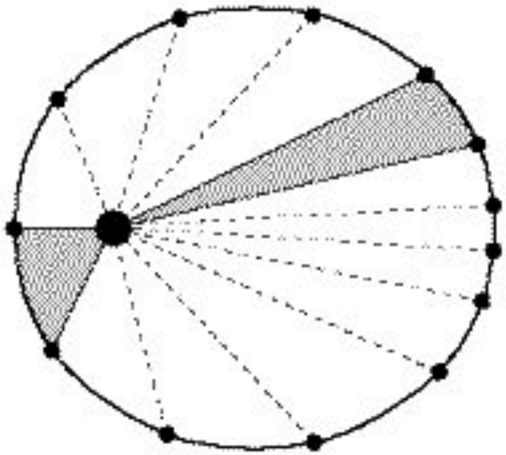
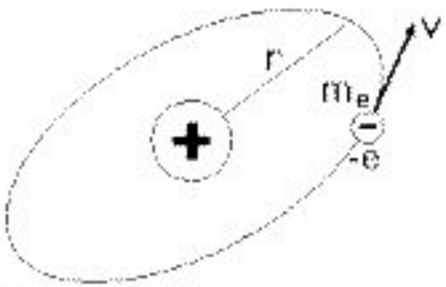
Calculating the magentic dipole moment of an orbiting electron



Kepler's Law of areas: *A line that connects a planet to the sun sweeps out equal areas in equal times.*

$$\left|\vec{L}\right| = mvr = m\underbrace{\left(\frac{2\pi r}{T}\right)}_{v} r = \frac{2m}{T}\underbrace{\left(\pi r^2\right)}_{A} = 2m\left(\frac{A}{T}\right)$$

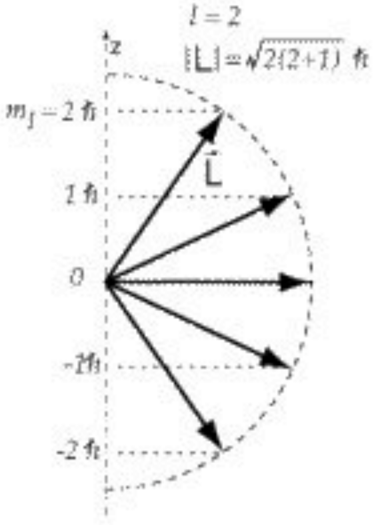


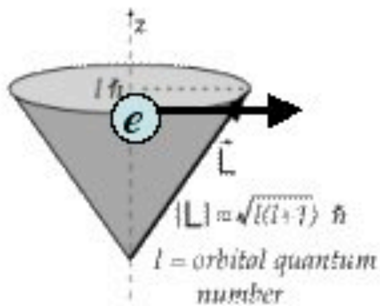
T = period of orbit

L = orbital angular momentum

$$\frac{A}{T} = \frac{\left|\vec{L}\right|}{2m} \qquad i = \frac{q}{T}$$

$$\vec{\mu} = iA = \frac{q}{2m}\vec{L} = -\frac{e}{2m}\vec{L}$$





Remember that the z component of angular momentum is quantized in units of h so the magnetic dipole moment is quantized as well:

$$\mu_z = -\frac{e}{2m_e}L_z = -\frac{eh}{2m_e}m_l = -\mu_B m_l$$

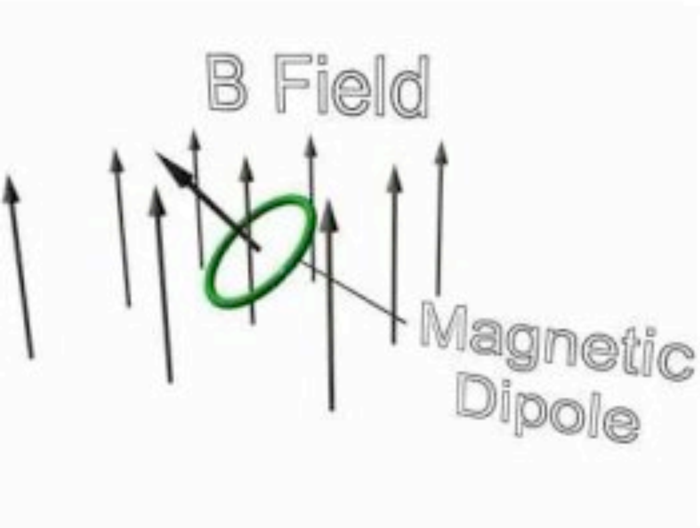
$$\frac{d^2\Phi}{d\phi^2} = -m_l^2\Phi(\phi)$$

where

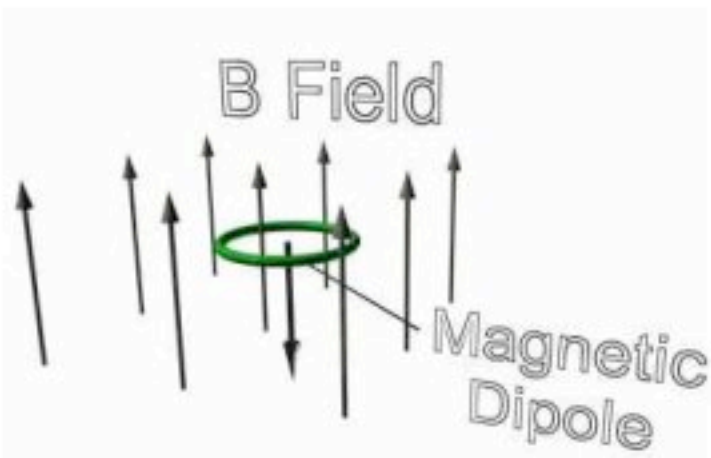
$$\mu_B = \frac{eh}{2m} = 9.274\times10^{-24}\,\mathrm{J/T}$$

The Bohr magneton

Dipole in a Magnetic Field



Magnetic dipole tends to want to align itself with the magnetic field but it can never align due to the uncertainty principle!

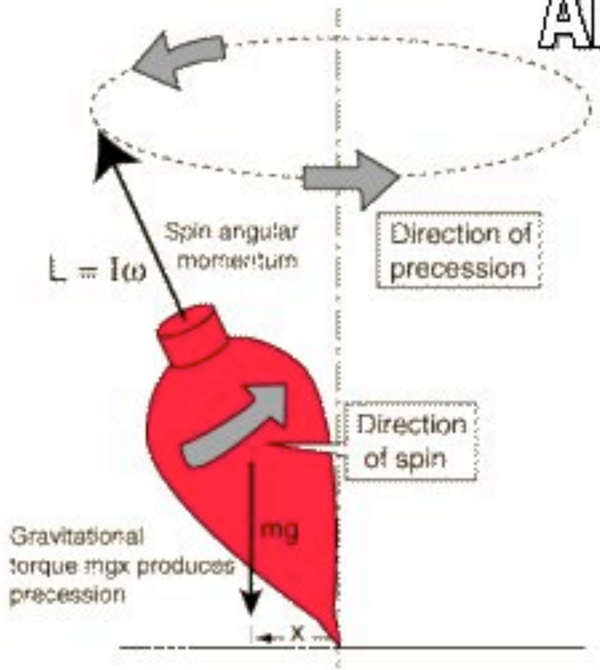


Torque exerted:

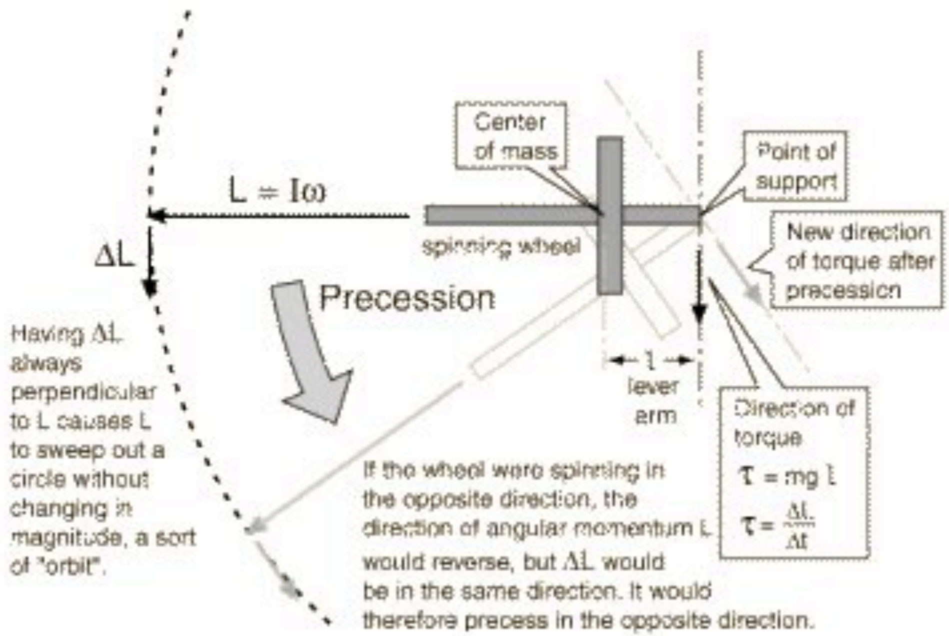
$$\vec{\tau} = \vec{\mu} \times B$$

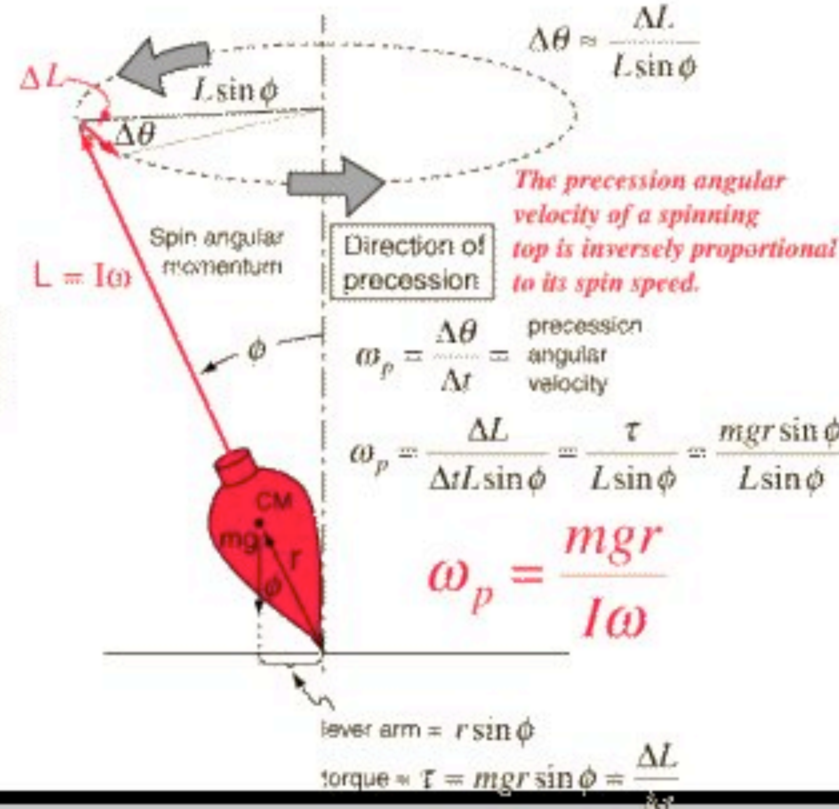
$$= \frac{\Delta L}{\Delta t}$$

Analogy: Precession of a spinning top

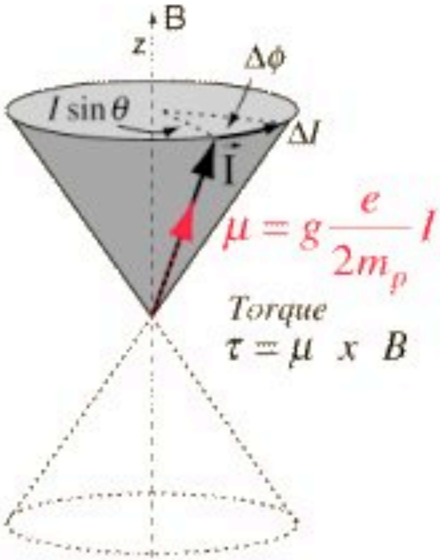


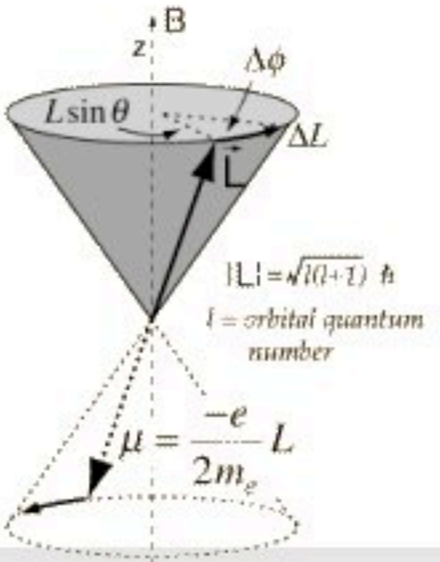
Here, the gravitational force provides the torque in place of a magnetic field and the angular momentum comes from the spinning of the top.





Precession of electron: Larmor frequency





$$L \sin\theta \cdot d\phi = \left|d\vec{L}\right|$$

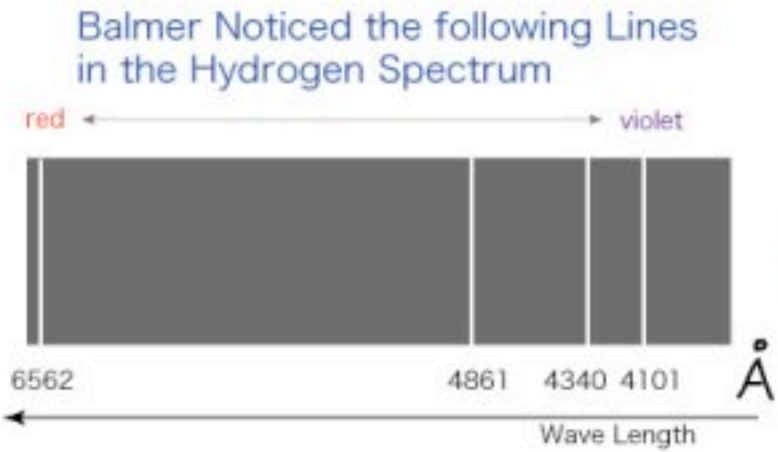
$$\left|d\vec{L}\right| = \left|\vec{\tau}\right| dt = \left|\frac{q}{2m_e} LB \sin\theta\right| dt$$

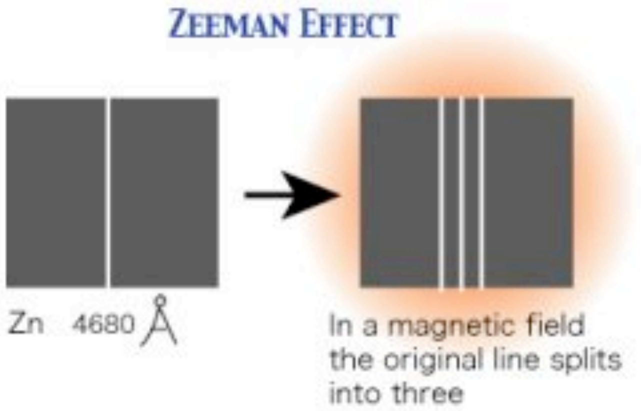
$$\omega_L = \frac{d\phi}{dt} = \frac{1}{L\sin\theta}\frac{\left|d\vec{L}\right|}{dt} = \frac{e}{2m_e} B$$

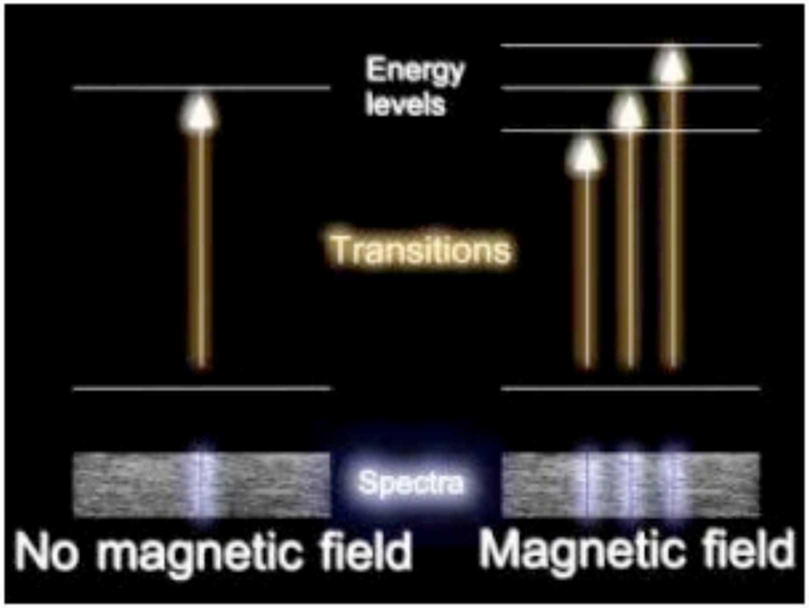
Zeeman effect

No magnetic field

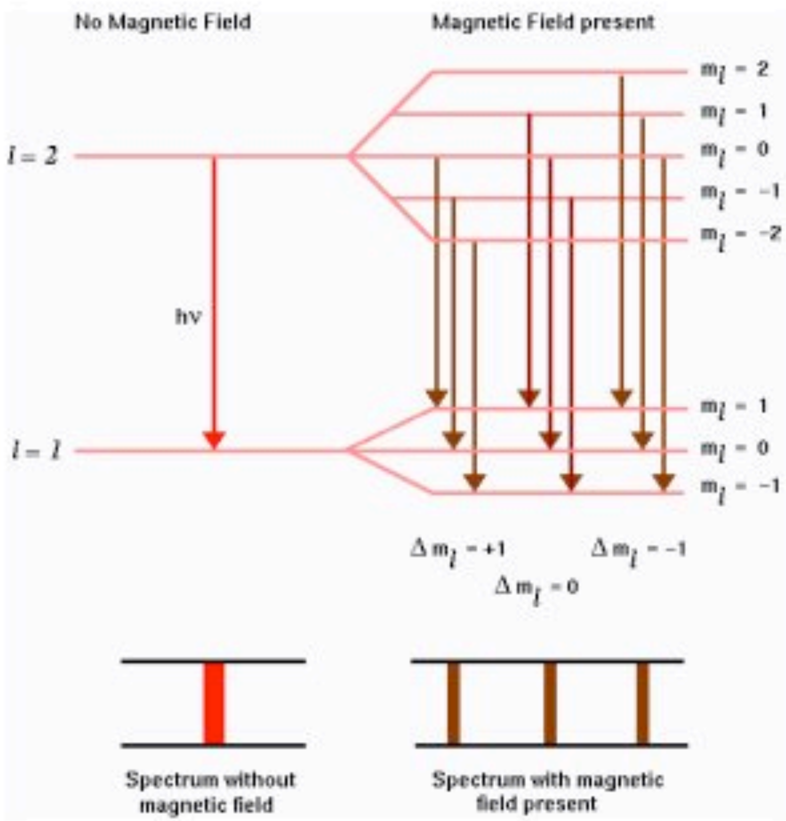
Magnetic field applied



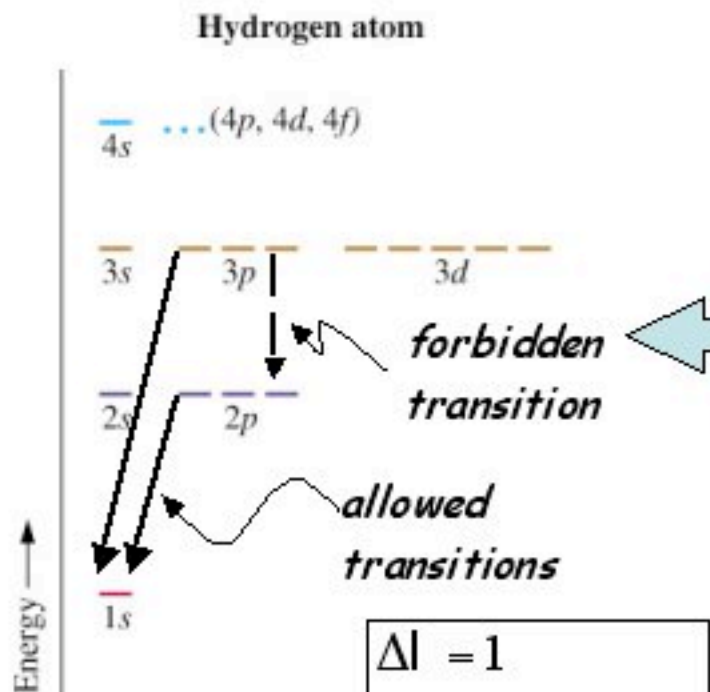




How many lines would we expect if the atom were placed in a magnetic field?



You expect a number of equally spaced "satellite lines" displaced from the emission lines by multiples of the Larmor frequency.



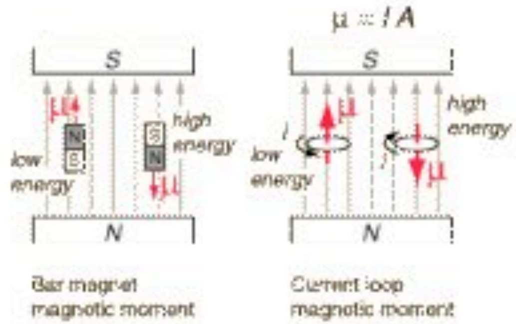
angular momentum must be conserved …photons carry angular momentum.

$$\Delta l = 1$$
$$\Delta m_l = -1, 0, +1$$

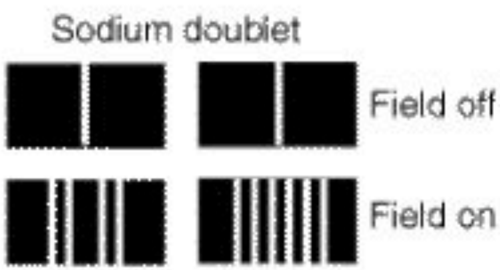
Remember that not all transitions are allowed. "Satellite" lines appear at the plus or minus the Larmor frequency only and not at multiples of that frequency.

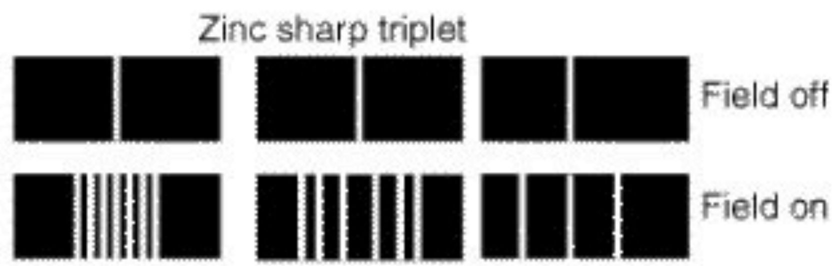
Electron orbits as a dipole moment

So we have seen that the current loop created by an electron orbiting in an atomic creates a dipole moment that interacts like a bar magnet with a magnetic field.



but wait, there's more

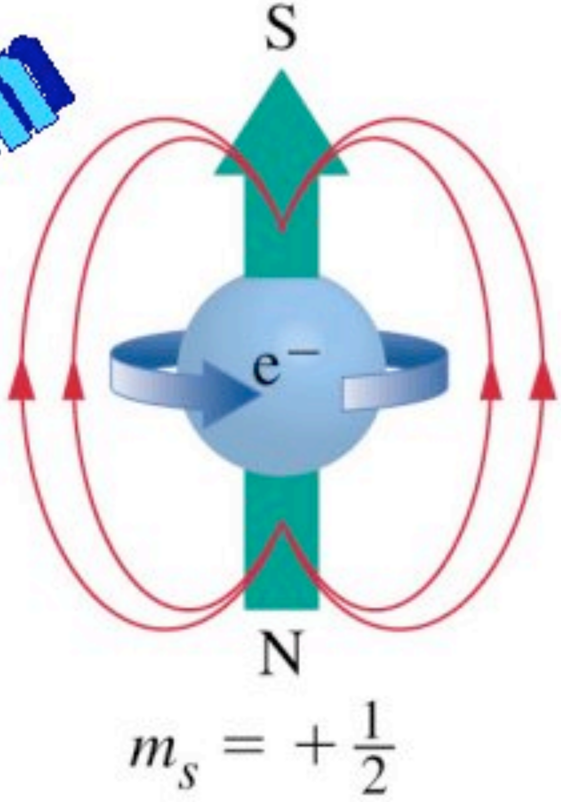


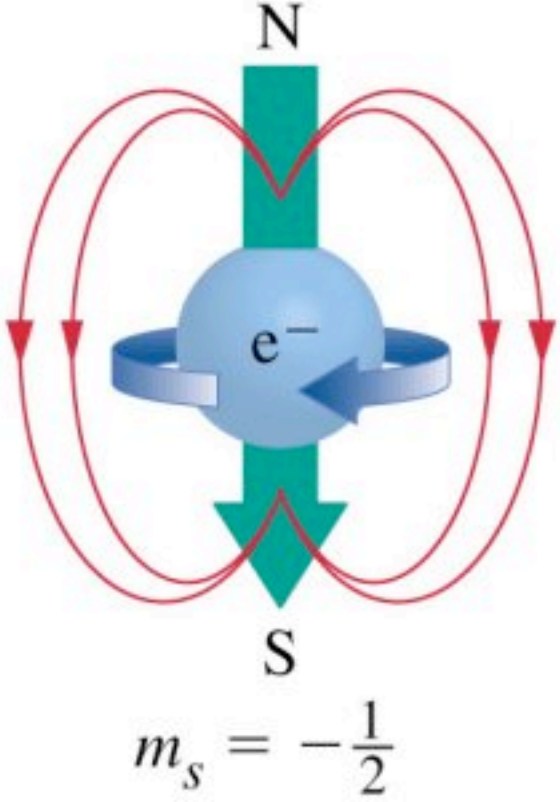


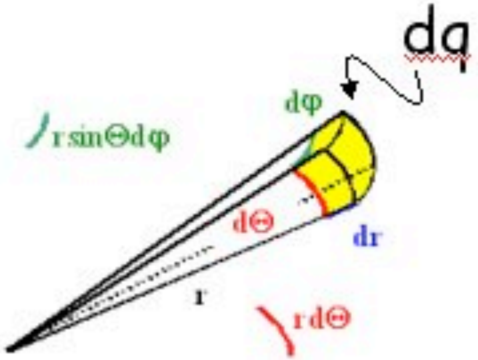
For many atoms, the number and spacing of the satellite lines are not what we would expect just from the orbital magnetic moment….there must be some other contribution to the magnetic moment.

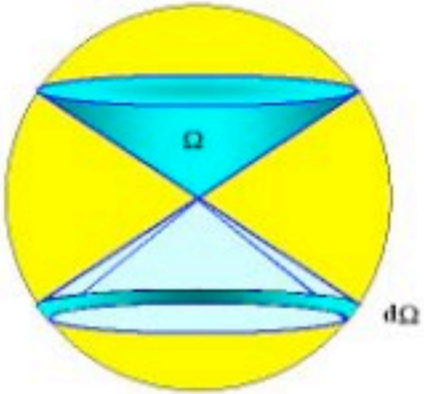
Electron spin

The electron has its own magnetic moment, and acts as a little bar magnet as well.



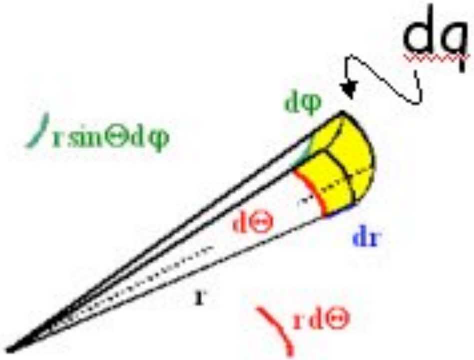






Classically: you could imagine a scenario where the electron had some volume and the charge were distributed uniformly throughout that volume such that if the electron spun on its axis, it would give rise to current loops.

The spin magnetic moment



In analogy to the orbital magnetic moment:

$$\vec{\mu} = iA = \frac{q}{2m}\vec{L} = -\frac{e}{2m}\vec{L}$$

the magnetic moments contributed by a differential elements of charge can be summed to be:

the "spin" magnetic moment

$$\vec{\mu}_s = \frac{q}{2m_e}\sum \vec{L}_i = -\frac{e}{2m_e}\vec{S}$$

the "spin" angular momentum

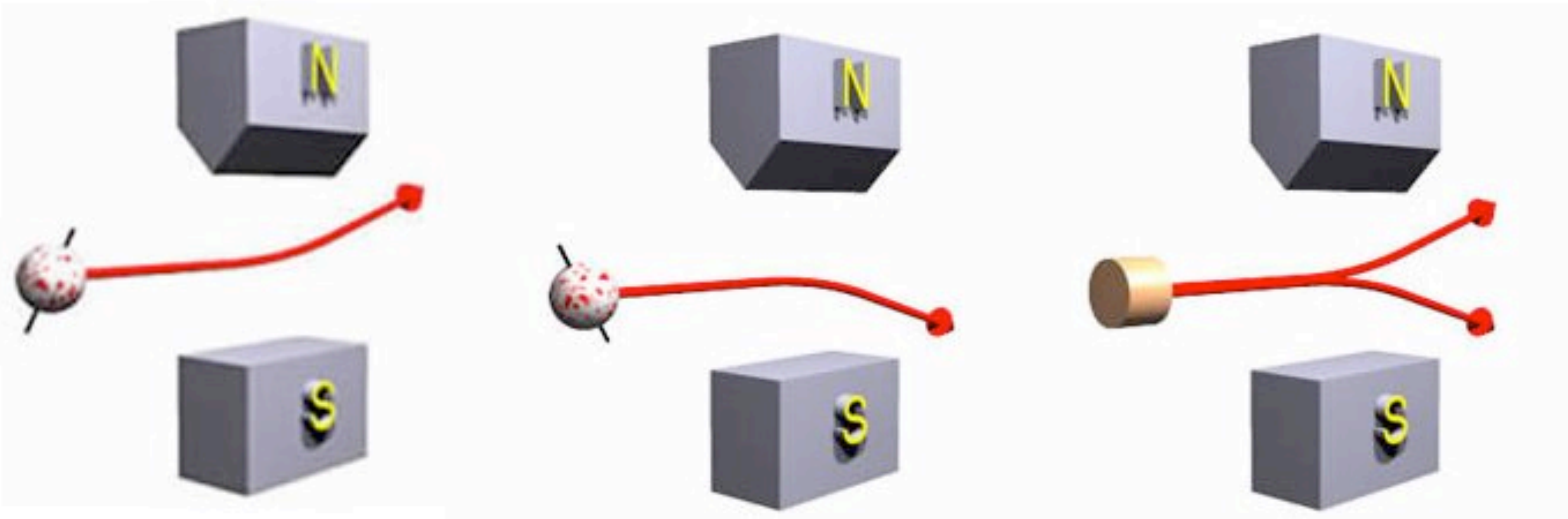
More generally, if the charge is not uniform:

$$\vec{\mu}_s = -g\frac{e}{2m_e}\vec{S}$$

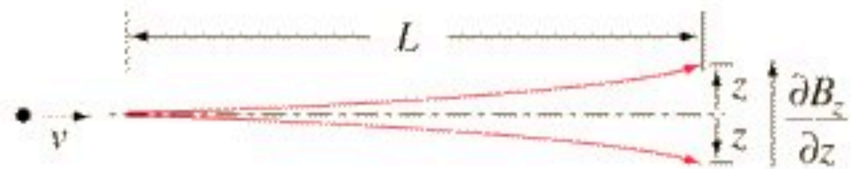
the "g" factor

$g = 2.002319304386$

Magnetic force on a spinning electron



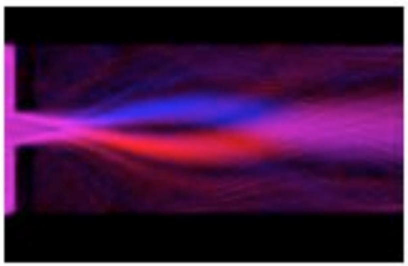
$$U = -\mu \cdot B = -\mu_B \frac{g}{2} B_z = \pm \mu_B B_z$$

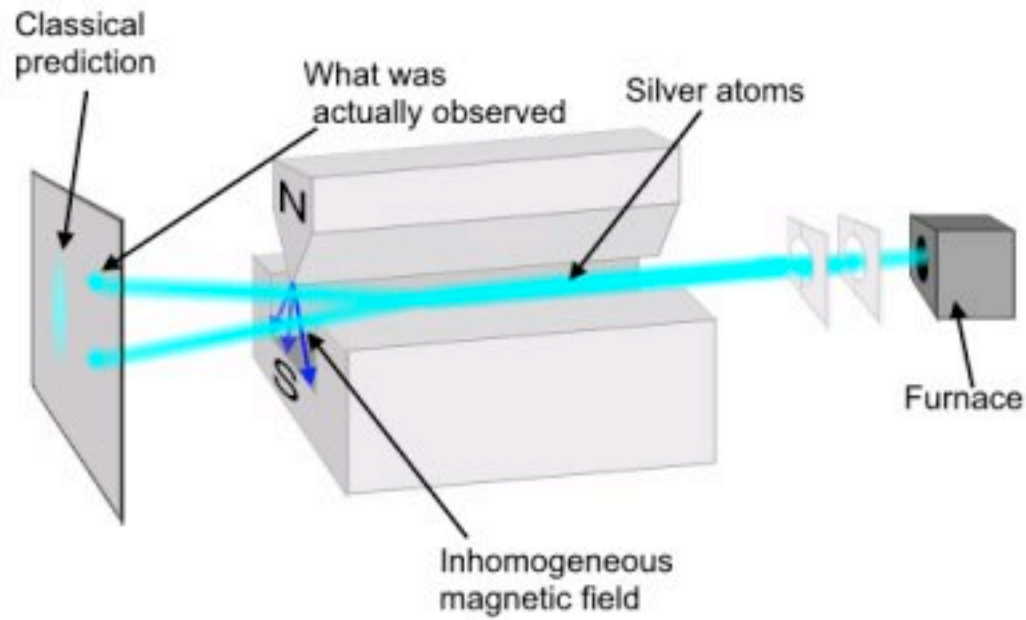


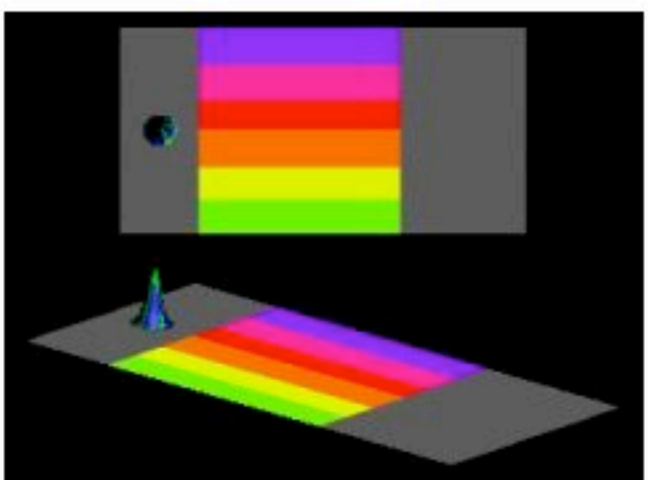
$$F_z = -\frac{\partial U}{\partial z} = \pm \mu_B \frac{\partial B_z}{\partial z}$$

$$z = \frac{1}{2} a t^2 = \frac{1}{2} \frac{F}{m} \left[\frac{L}{v} \right]^2 = \pm \frac{\mu_B L^2}{4KE} \frac{\partial B_z}{\partial z}$$

The Stern Gerlach experiment





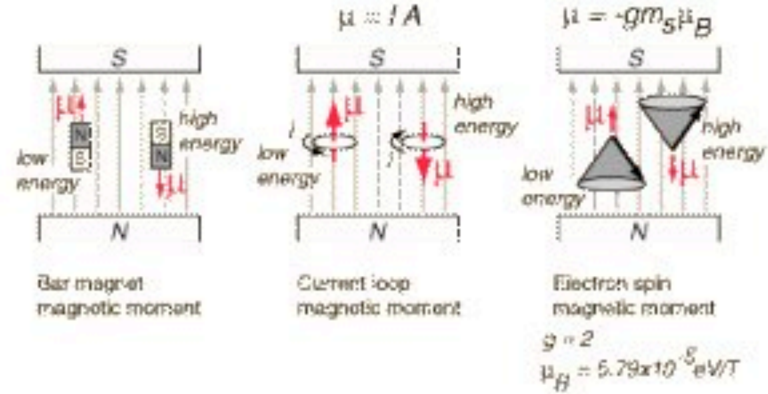


Beam split into two discrete parts! Outer electron in silver is in an s state (l=0), magnetic moment comes from the spin of the outer electron.

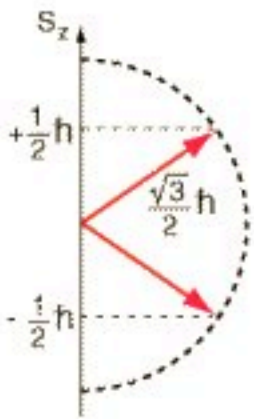
Spin contribution to the magnetic moment

In addition to the orbital magnetic moment, we must take into account the spin.

$$\vec{\mu} = \vec{\mu}_0 + \vec{\mu}_s = \frac{-e}{2m_e}\left\{\vec{L} + g\vec{S}\right\}$$



The spin orientation:



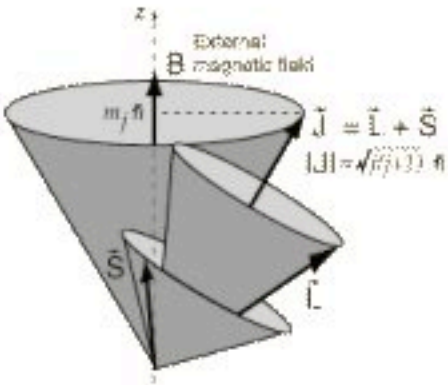
Electrons come in “spin up” and “spin down” states.

$$S_z = m_s \hbar \quad \text{where } m_s = \frac{1}{2} \text{ or } -\frac{1}{2}$$

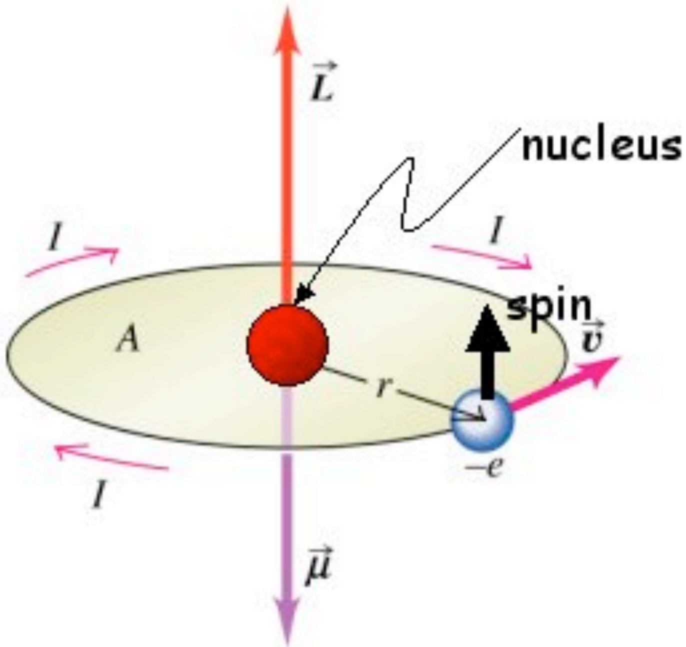
The magnitude of the spin angular momentum is:

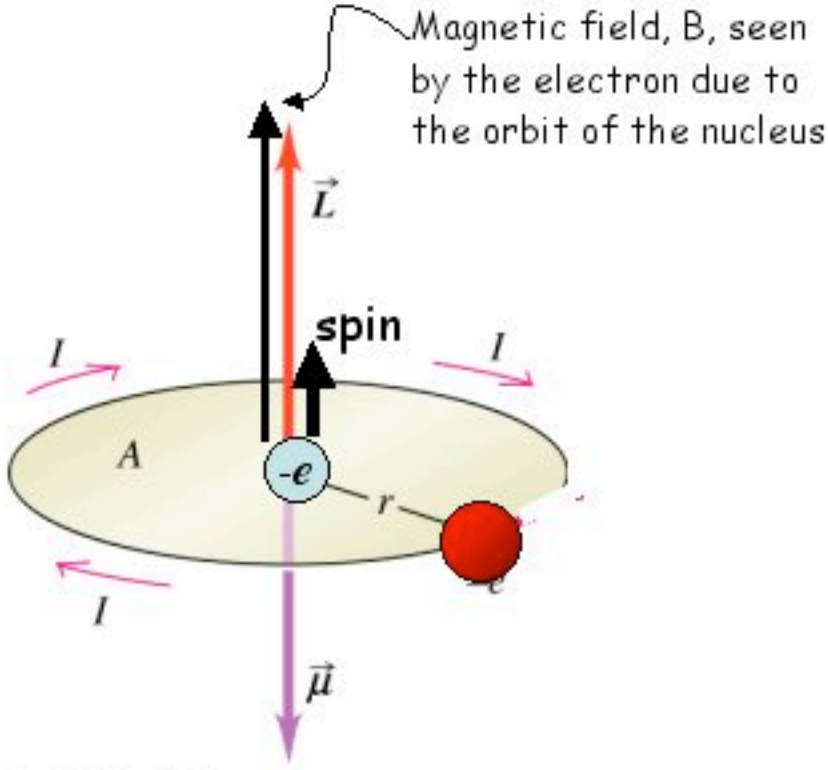
$$\left|\vec{S}\right| = \sqrt{s(s+1)}\,\hbar = \frac{\sqrt{3}}{2}\hbar$$

The Internal Magnetic field

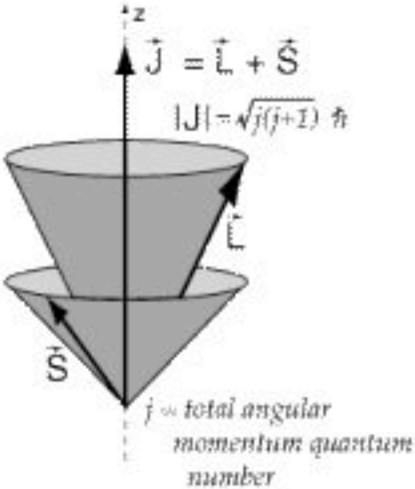


We have learned about how an external magnetic field interacts with the magnetic moments in the atom, but if we look at this from the point of view of an electron, we realize that the electron "sees" a magnetic field from the apparent orbit of the positively charged nucleus.





Spin Orbit Interaction



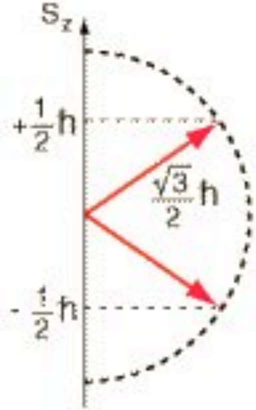
$$\vec{J} = \vec{L} + \vec{S}$$

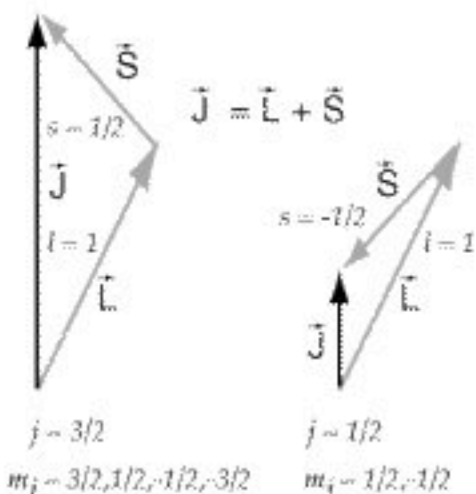
$$|\vec{J}| = \sqrt{j(j+1)}\,\hbar$$

$$J_z = m_j \hbar \quad \text{with } m_j = j, j-1, \ldots, -j$$

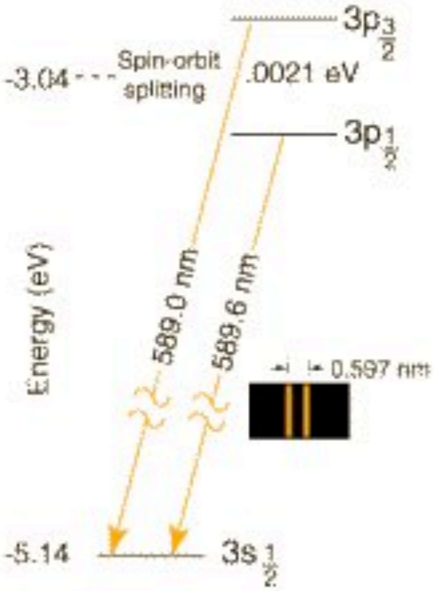
total angular momentum quantum number :

$$j = l + s,\ \ l + s - 1, \ldots, |l + s|$$





Sodium Doublet



Spectrosopic Notation

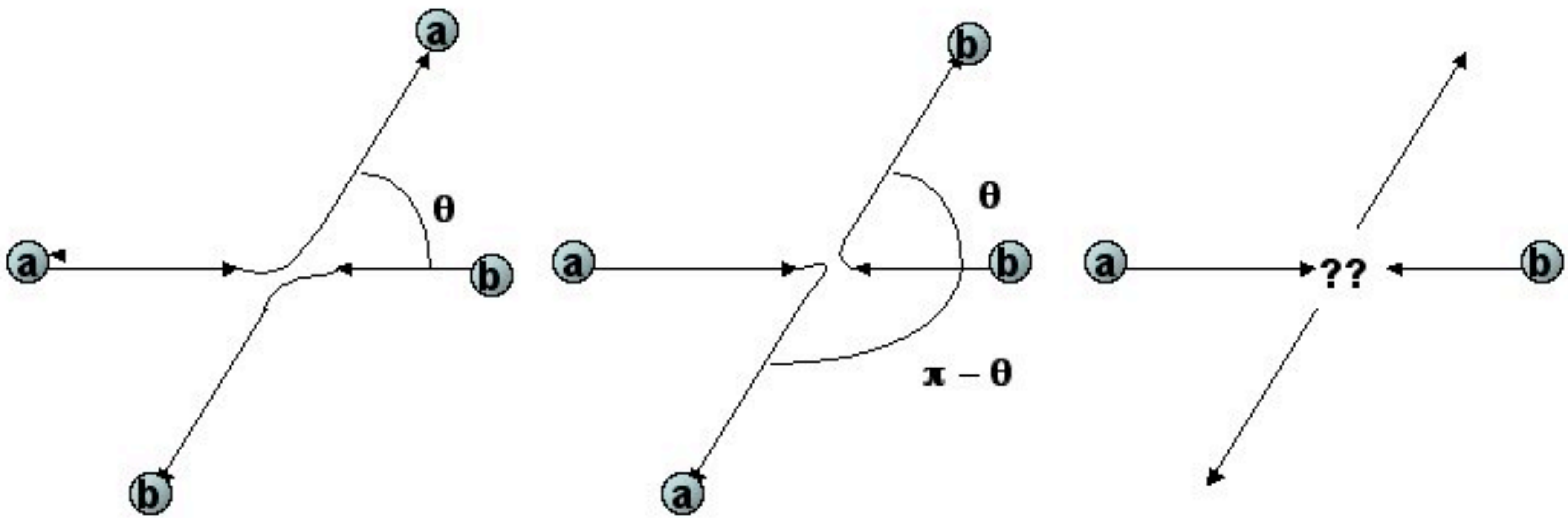
$$^{2S+1}L_J$$

Term Symbol

			n=1	n=2	n=3	n=4
s	-- sharp	$\ell=0$	1s	2s	3s	4s
p	-- principal	$\ell=1$		2p	3p	4p
d	-- diffuse	$\ell=2$			3d	4d
f	-- fundamental	$\ell=3$				4f
g		$\ell=4$	beyond this point, the notation			
h		$\ell=5$	just follows the alphabet			
⋮						

Indistinguishability of particles

The first two pictures give the same outcome. Even though a and b are identical, you can tell them apart by following them along their unique paths.



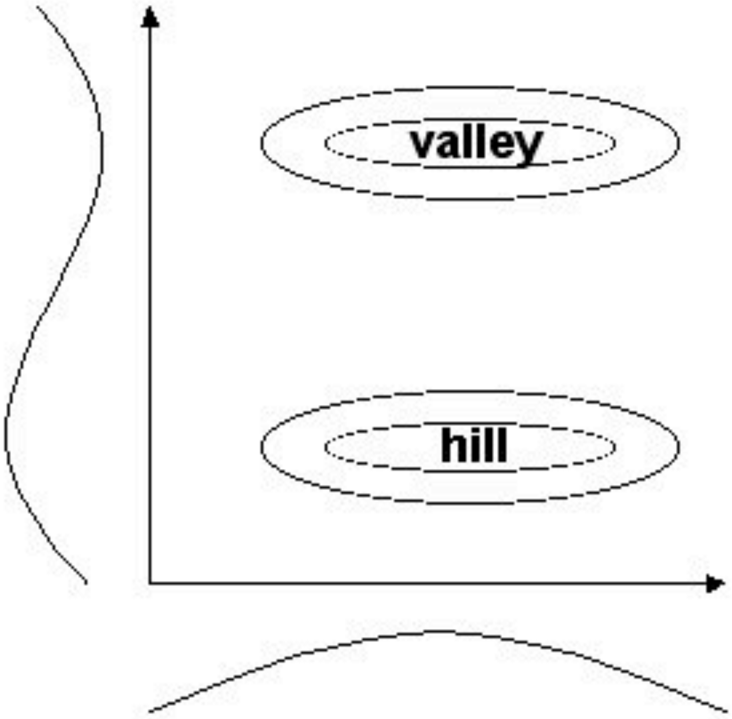
Quantum mechanically, each particle has some probability of being somewhere at a particular time, which overlaps greatly at the collision point.

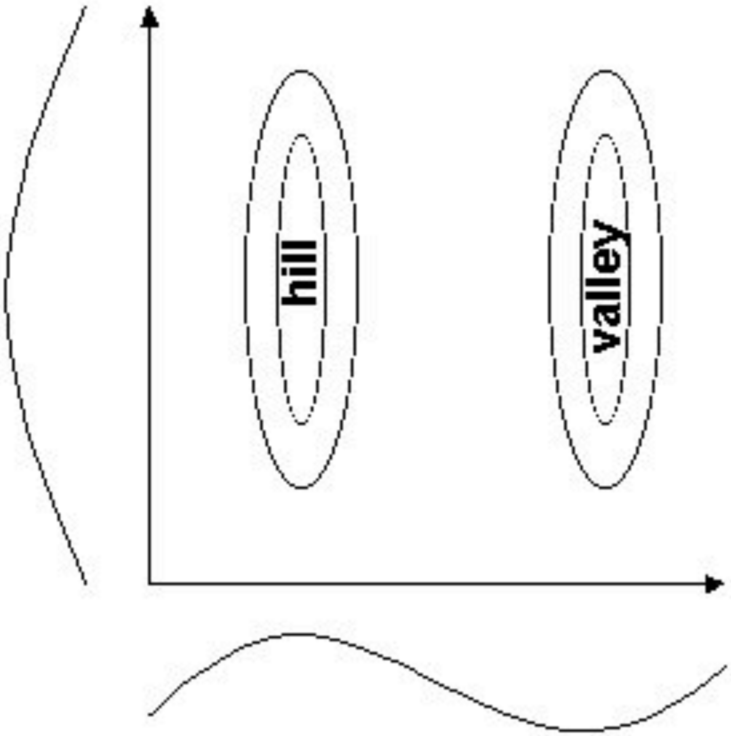
Which particle emerges where? In wave terms, they interfered.

Probability

Consider two particles in a box, one in the n=1 state, the other in the n=2 state.

$$\Psi_A(x1)\,\Psi_B(x2) \qquad \Psi_A(x2)\,\Psi_B(x1)$$

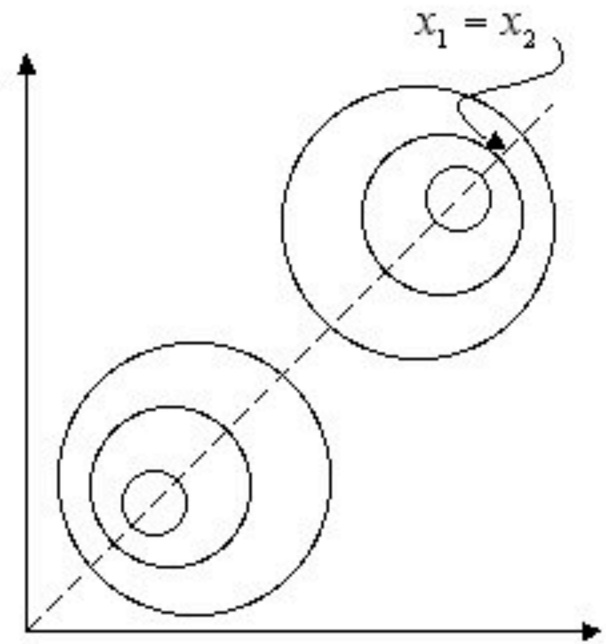




Wavefunction not generally symmetric under exchange of identical particles!!

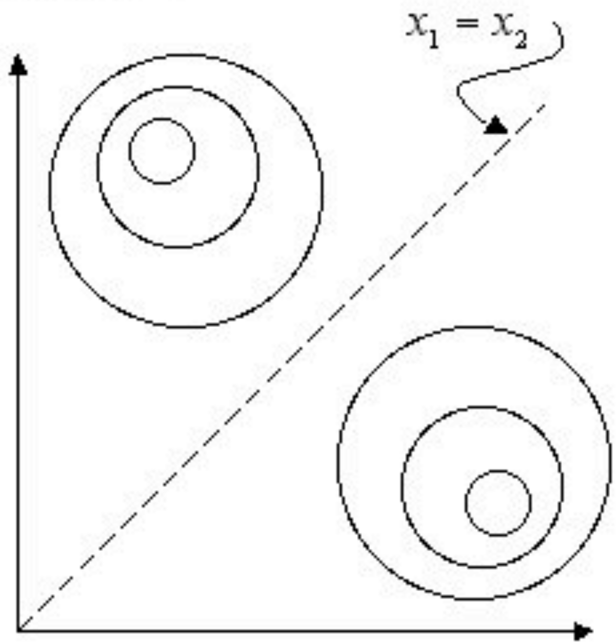
Symmetric and Antisymmetric Wavefunctions

Symmetric: probability generally highest when particles are closest together. "Huddling".



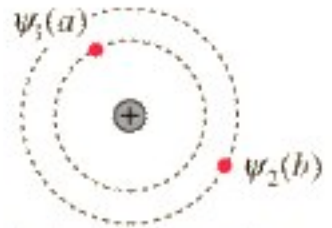
$$\psi_S(x_1, x_2) = \frac{1}{\sqrt{2}}\left[\psi_A(x_1) \bullet \psi_B(x_2) + \psi_A(x_2) \bullet \psi_B(x_1)\right]$$

Antisymmetric: probability generally highest when particles are furthest apart. "avoiding one another".



$$\psi_S(x_1, x_2) = \frac{1}{\sqrt{2}}\left[\psi_A(x_1) \bullet \psi_B(x_2) - \psi_A(x_2) \bullet \psi_B(x_1)\right]$$

The Pauli Exclusion Principle



For fermions the negative sign must be used, so that the wavefunction goes to identically zero if the states a and b are identical.