

# Table of Integrals

## BASIC FORMS

- (1)  $\int x^n dx = \frac{1}{n+1} x^{n+1}$
- (2)  $\int \frac{1}{x} dx = \ln x$
- (3)  $\int u dv = uv - \int v du$
- (4)  $\int u(x)v'(x) dx = u(x)v(x) - \int v(x)u'(x) dx$

## RATIONAL FUNCTIONS

- (5)  $\int \frac{1}{ax+b} dx = \frac{1}{a} \ln(ax+b)$
- (6)  $\int \frac{1}{(x+a)^2} dx = \frac{-1}{x+a}$
- (7)  $\int (x+a)^n dx = (x+a)^n \left( \frac{a}{1+n} + \frac{x}{1+n} \right), n \neq -1$
- (8)  $\int x(x+a)^n dx = \frac{(x+a)^{1+n}(nx+x-a)}{(n+2)(n+1)}$
- (9)  $\int \frac{dx}{1+x^2} = \tan^{-1} x$
- (10)  $\int \frac{dx}{a^2+x^2} = \frac{1}{a} \tan^{-1}(x/a)$
- (11)  $\int \frac{x dx}{a^2+x^2} = \frac{1}{2} \ln(a^2+x^2)$
- (12)  $\int \frac{x^2 dx}{a^2+x^2} = x - a \tan^{-1}(x/a)$
- (13)  $\int \frac{x^3 dx}{a^2+x^2} = \frac{1}{2} x^2 - \frac{1}{2} a^2 \ln(a^2+x^2)$
- (14)  $\int (ax^2+bx+c)^{-1} dx = \frac{2}{\sqrt{4ac-b^2}} \tan^{-1} \left( \frac{2ax+b}{\sqrt{4ac-b^2}} \right)$
- (15)  $\int \frac{1}{(x+a)(x+b)} dx = \frac{1}{b-a} [\ln(a+x) - \ln(b+x)], a \neq b$
- (16)  $\int \frac{x}{(x+a)^2} dx = \frac{a}{a+x} + \ln(a+x)$
- (17)  $\int \frac{x}{ax^2+bx+c} dx = \frac{\ln(ax^2+bx+c)}{2a} - \frac{b}{a\sqrt{4ac-b^2}} \tan^{-1} \left( \frac{2ax+b}{\sqrt{4ac-b^2}} \right)$

## INTEGRALS WITH ROOTS

- (18)  $\int \sqrt{x-a} dx = \frac{2}{3} (x-a)^{3/2}$
- (19)  $\int \frac{1}{\sqrt{x \pm a}} dx = 2\sqrt{x \pm a}$
- (20)  $\int \frac{1}{\sqrt{a-x}} dx = 2\sqrt{a-x}$
- (21)  $\int x\sqrt{x-a} dx = \frac{2}{3} a(x-a)^{3/2} + \frac{2}{5} (x-a)^{5/2}$
- (22)  $\int \sqrt{ax+b} dx = \left( \frac{2b}{3a} + \frac{2x}{3} \right) \sqrt{b+ax}$
- (23)  $\int (ax+b)^{3/2} dx = \sqrt{b+ax} \left( \frac{2b^2}{5a} + \frac{4bx}{5} + \frac{2ax^2}{5} \right)$
- (24)  $\int \frac{x}{\sqrt{x \pm a}} dx = \frac{2}{3} (x \pm 2a) \sqrt{x \pm a}$
- (25)  $\int \sqrt{\frac{x}{a-x}} dx = -\sqrt{x}\sqrt{a-x} - a \tan^{-1} \left( \frac{\sqrt{x}\sqrt{a-x}}{x-a} \right)$
- (26)  $\int \sqrt{\frac{x}{x+a}} dx = \sqrt{x}\sqrt{x+a} - a \ln[\sqrt{x} + \sqrt{x+a}]$
- (27)  $\int x\sqrt{ax+b} dx = \left( -\frac{4b^2}{15a^2} + \frac{2bx}{15a} + \frac{2x^2}{5} \right) \sqrt{b+ax}$
- (28)  $\int \sqrt{x}\sqrt{ax+b} dx = \left( \frac{b\sqrt{x}}{4a} + \frac{x^{3/2}}{2} \right) \sqrt{b+ax} - \frac{b^2 \ln(2\sqrt{a}\sqrt{x} + 2\sqrt{b+ax})}{4a^{3/2}}$
- (29)  $\int x^{3/2} \sqrt{ax+b} dx = \left( -\frac{b^2\sqrt{x}}{8a^2} + \frac{bx^{3/2}}{12a} + \frac{x^{5/2}}{3} \right) \sqrt{b+ax} - \frac{b^3 \ln(2\sqrt{a}\sqrt{x} + 2\sqrt{b+ax})}{8a^{5/2}}$
- (30)  $\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2} x\sqrt{x^2 \pm a^2} \pm \frac{1}{2} a^2 \ln(x + \sqrt{x^2 \pm a^2})$
- (31)  $\int \sqrt{a^2 - x^2} dx = \frac{1}{2} x\sqrt{a^2 - x^2} - \frac{1}{2} a^2 \tan^{-1} \left( \frac{x\sqrt{a^2 - x^2}}{x^2 - a^2} \right)$
- (32)  $\int x\sqrt{x^2 \pm a^2} = \frac{1}{3} (x^2 \pm a^2)^{3/2}$
- (33)  $\int \frac{1}{\sqrt{x^2 \pm a^2}} dx = \ln(x + \sqrt{x^2 \pm a^2})$

$$(34) \int \frac{1}{\sqrt{a^2 - x^2}} = \sin^{-1} \frac{x}{a}$$

$$(35) \int \frac{x}{\sqrt{x^2 \pm a^2}} = \sqrt{x^2 \pm a^2}$$

$$(36) \int \frac{x}{\sqrt{a^2 - x^2}} dx = -\sqrt{a^2 - x^2}$$

$$(37) \int \frac{x^2}{\sqrt{x^2 \pm a^2}} dx = \frac{1}{2} x \sqrt{x^2 \pm a^2} \mp \frac{1}{2} \ln \left( x + \sqrt{x^2 \pm a^2} \right)$$

$$(38) \int \frac{x^2}{\sqrt{a^2 - x^2}} dx = -\frac{1}{2} x \sqrt{a^2 - x^2} - \frac{1}{2} a^2 \tan^{-1} \left( \frac{x \sqrt{a^2 - x^2}}{x^2 - a^2} \right)$$

$$(39) \int \sqrt{ax^2 + bx + c} dx = \left( \frac{b}{4a} + \frac{x}{2} \right) \sqrt{ax^2 + bx + c} + \frac{4ac - b^2}{8a^{3/2}} \ln \left( \frac{2ax + b}{\sqrt{a}} + 2\sqrt{ax^2 + bx + c} \right)$$

$$(40) \int x \sqrt{ax^2 + bx + c} dx = \left( \frac{x^3}{3} + \frac{bx}{12a} + \frac{8ac - 3b^2}{24a^2} \right) \sqrt{ax^2 + bx + c} - \frac{b(4ac - b^2)}{16a^{5/2}} \ln \left( \frac{2ax + b}{\sqrt{a}} + 2\sqrt{ax^2 + bx + c} \right)$$

$$(41) \int \frac{1}{\sqrt{ax^2 + bx + c}} dx = \frac{1}{\sqrt{a}} \ln \left[ \frac{2ax + b}{\sqrt{a}} + 2\sqrt{ax^2 + bx + c} \right]$$

$$(42) \int \frac{x}{\sqrt{ax^2 + bx + c}} dx = \frac{1}{a} \sqrt{ax^2 + bx + c} - \frac{b}{2a^{3/2}} \ln \left[ \frac{2ax + b}{\sqrt{a}} + 2\sqrt{ax^2 + bx + c} \right]$$

### LOGARITHMS

$$(43) \int \ln x dx = x \ln x - x$$

$$(44) \int \frac{\ln(ax)}{x} dx = \frac{1}{2} (\ln(ax))^2$$

$$(45) \int \ln(ax + b) dx = \frac{ax + b}{a} \ln(ax + b) - x$$

$$(46) \int \ln(a^2 x^2 \pm b^2) dx = x \ln(a^2 x^2 \pm b^2) + \frac{2b}{a} \tan^{-1} \left( \frac{ax}{b} \right) - 2x$$

$$(47) \int \ln(a^2 - b^2 x^2) dx = x \ln(a^2 - b^2 x^2) + \frac{2a}{b} \tan^{-1} \left( \frac{bx}{a} \right) - 2x$$

$$(48) \int \ln(ax^2 + bx + c) dx = \frac{1}{a} \sqrt{4ac - b^2} \tan^{-1} \left( \frac{2ax + b}{\sqrt{4ac - b^2}} \right) - 2x + \left( \frac{b}{2a} + x \right) \ln(ax^2 + bx + c)$$

$$(49) \int x \ln(ax + b) dx = \frac{b}{2a} x - \frac{1}{4} x^2 + \frac{1}{2} \left( x^2 - \frac{b^2}{a^2} \right) \ln(ax + b)$$

$$(50) \int x \ln(a^2 - b^2 x^2) dx = -\frac{1}{2} x^2 + \frac{1}{2} \left( x^2 - \frac{a^2}{b^2} \right) \ln(a^2 - b^2 x^2)$$

### EXPONENTIALS

$$(51) \int e^{ax} dx = \frac{1}{a} e^{ax}$$

$$(52) \int \sqrt{x} e^{ax} dx = \frac{1}{a} \sqrt{x} e^{ax} + \frac{i\sqrt{\pi}}{2a^{3/2}} \operatorname{erf}(i\sqrt{ax}) \quad \text{where}$$

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$$

$$(53) \int x e^x dx = (x - 1) e^x$$

$$(54) \int x e^{ax} dx = \left( \frac{x}{a} - \frac{1}{a^2} \right) e^{ax}$$

$$(55) \int x^2 e^x dx = e^x (x^2 - 2x + 2)$$

$$(56) \int x^2 e^{ax} dx = e^{ax} \left( \frac{x^2}{a} - \frac{2x}{a^2} + \frac{2}{a^3} \right)$$

$$(57) \int x^3 e^x dx = e^x (x^3 - 3x^2 + 6x - 6)$$

$$(58) \int x^n e^{ax} dx = (-1)^n \frac{1}{a} \Gamma[1 + n, -ax] \quad \text{where}$$

$$\Gamma(a, x) = \int_x^\infty t^{a-1} e^{-t} dt$$

$$(59) \int e^{ax^2} dx = -i \frac{\sqrt{\pi}}{2\sqrt{a}} \operatorname{erf}(ix\sqrt{a})$$

### TRIGONOMETRIC FUNCTIONS

$$(60) \int \sin x dx = -\cos x$$

$$(61) \int \sin^2 x dx = \frac{x}{2} - \frac{1}{4} \sin 2x$$

$$(62) \int \sin^3 x dx = -\frac{3}{4} \cos x + \frac{1}{12} \cos 3x$$

$$(63) \int \cos x dx = \sin x$$

$$(64) \int \cos^2 x dx = \frac{x}{2} + \frac{1}{4} \sin 2x$$

$$(65) \int \cos^3 x dx = \frac{3}{4} \sin x + \frac{1}{12} \sin 3x$$

$$(66) \int \sin x \cos x dx = -\frac{1}{2} \cos^2 x$$

$$(67) \int \sin^2 x \cos x dx = \frac{1}{4} \sin x - \frac{1}{12} \sin 3x$$

$$(68) \int \sin x \cos^2 x dx = -\frac{1}{4} \cos x - \frac{1}{12} \cos 3x$$

$$(69) \int \sin^2 x \cos^2 x dx = \frac{x}{8} - \frac{1}{32} \sin 4x$$

$$(70) \int \tan x dx = -\ln |\cos x|$$

$$(71) \int \tan^2 x dx = -x + \tan x$$

$$(72) \int \tan^3 x dx = \ln |\cos x| + \frac{1}{2} \sec^2 x$$

$$(73) \int \sec x dx = \ln |\sec x + \tan x|$$

$$(74) \int \sec^2 x dx = \tan x$$

$$(75) \int \sec^3 x dx = \frac{1}{2} \sec x \tan x + \frac{1}{2} \ln |\sec x \tan x|$$

$$(76) \int \sec x \tan x dx = \sec x$$

$$(77) \int \sec^2 x \tan x dx = \frac{1}{2} \sec^2 x$$

$$(78) \int \sec^n x \tan x dx = \frac{1}{n} \sec^n x, \quad n \neq 0$$

$$(79) \int \csc x dx = \ln |\csc x - \cot x|$$

$$(80) \int \csc^2 x dx = -\cot x$$

$$(81) \int \csc^3 x dx = -\frac{1}{2} \cot x \csc x + \frac{1}{2} \ln |\csc x - \cot x|$$

$$(82) \int \csc^n x \cot x dx = -\frac{1}{n} \csc^n x, \quad n \neq 0$$

$$(83) \int \sec x \csc x dx = \ln \tan x$$

#### TRIGONOMETRIC FUNCTIONS WITH $x^n$

$$(84) \int x \cos x dx = \cos x + x \sin x$$

$$(85) \int x \cos(ax) dx = \frac{1}{a^2} \cos ax + \frac{1}{a} x \sin ax$$

$$(86) \int x^2 \cos x dx = 2x \cos x + (x^2 - 2) \sin x$$

$$(87) \int x^2 \cos ax dx = \frac{2}{a^2} x \cos ax + \frac{a^2 x^2 - 2}{a^3} \sin ax$$

$$(88) \int x^n \cos x dx = -\frac{1}{2} (i)^{1+n} [\Gamma(1+n, -ix) + (-1)^n \Gamma(1+n, ix)]$$

$$(89) \int x^n \cos ax dx = \frac{1}{2} (ia)^{1-n} [(-1)^n \Gamma(1+n, -iax) - \Gamma(1+n, iax)]$$

$$(90) \int x \sin x dx = -x \cos x + \sin x$$

$$(91) \int x \sin(ax) dx = -\frac{x}{a} \cos ax + \frac{1}{a^2} \sin ax$$

$$(92) \int x^2 \sin x dx = (2 - x^2) \cos x + 2x \sin x$$

$$(93) \int x^2 \sin ax dx = \frac{2 - a^2 x^2}{a^3} \cos ax + \frac{2}{a^3} x \sin ax$$

$$(94) \int x^n \sin x dx = -\frac{1}{2} (i)^n [\Gamma(n+1, -ix) - (-1)^n \Gamma(n+1, -ix)]$$

#### TRIGONOMETRIC FUNCTIONS WITH $e^{ax}$

$$(95) \int e^x \sin x dx = \frac{1}{2} e^x [\sin x - \cos x]$$

$$(96) \int e^{bx} \sin(ax) dx = \frac{1}{b^2 + a^2} e^{bx} [b \sin ax - a \cos ax]$$

$$(97) \int e^x \cos x dx = \frac{1}{2} e^x [\sin x + \cos x]$$

$$(98) \int e^{bx} \cos(ax) dx = \frac{1}{b^2 + a^2} e^{bx} [a \sin ax + b \cos ax]$$

#### TRIGONOMETRIC FUNCTIONS WITH $x^n$ AND $e^{ax}$

$$(99) \int x e^x \sin x dx = \frac{1}{2} e^x [\cos x - x \cos x + x \sin x]$$

$$(100) \int x e^x \cos x dx = \frac{1}{2} e^x [x \cos x - \sin x + x \sin x]$$

#### HYPERBOLIC FUNCTIONS

$$(101) \int \cosh x dx = \sinh x$$

$$(102) \int e^{ax} \cosh bxdx = \frac{e^{ax}}{a^2 - b^2} [a \cosh bx - b \sinh bx]$$

$$(103) \int \sinh x dx = \cosh x$$

$$(104) \int e^{ax} \sinh bxdx = \frac{e^{ax}}{a^2 - b^2} [-b \cosh bx + a \sinh bx]$$

$$(105) \int e^x \tanh x dx = e^x - 2 \tan^{-1}(e^x)$$

$$(106) \int \tanh ax dx = \frac{1}{a} \ln \cosh ax$$

$$(107) \int \cos ax \cosh bxdx = \frac{1}{a^2 + b^2} [a \sin ax \cosh bx + b \cos ax \sinh bx]$$

$$(108) \quad \int \cos ax \sinh bxdx = \frac{1}{a^2 + b^2} [b \cos ax \cosh bx + a \sin ax \sinh bx]$$

$$(109) \quad \int \sin ax \cosh bxdx = \frac{1}{a^2 + b^2} [-a \cos ax \cosh bx + b \sin ax \sinh bx]$$

$$(110) \quad \int \sin ax \sinh bxdx = \frac{1}{a^2 + b^2} [b \cosh bx \sin ax - a \cos ax \sinh bx]$$

$$(111) \quad \int \sinh ax \cosh ax dx = \frac{1}{4a} [-2ax + \sinh(2ax)]$$

$$(112) \quad \int \sinh ax \cosh bxdx = \frac{1}{b^2 - a^2} [b \cosh bx \sinh ax - a \cosh ax \sinh bx]$$