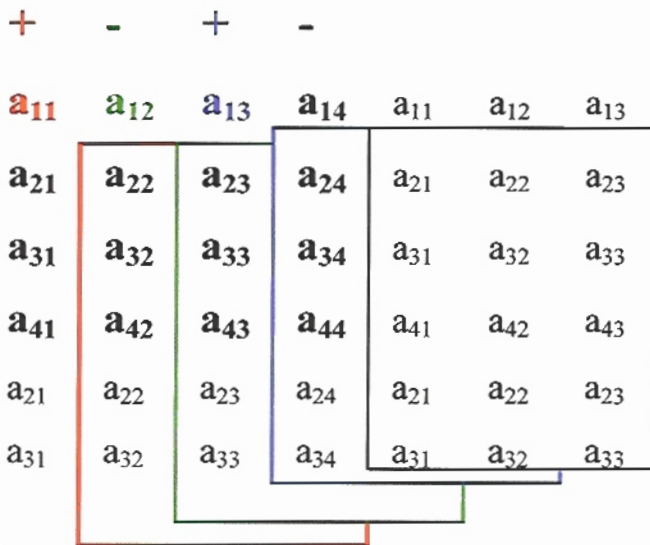Can we determine the determinant of a matrix (4x4) without using the co-factor method? Yes, this method will explain step by step on how use the diagonal method & co-factor to determine the matrix 4x4 determinant.

1- Copy the 2nd & 3rd row below 4th row respectively.
2- Copy 1st, 2nd, & 3rd column next to the 4th column respectively as is shown below.

$$+ \quad - \quad + \quad -$$

a_{11}	a_{12}	a_{13}	a_{14}	a_{11}	a_{12}	a_{13}
a_{21}	a_{22}	a_{23}	a_{24}	a_{21}	a_{22}	a_{23}
a_{31}	a_{32}	a_{33}	a_{34}	a_{31}	a_{32}	a_{33}
a_{41}	a_{42}	a_{43}	a_{44}	a_{41}	a_{42}	a_{43}
a_{21}	a_{22}	a_{23}	a_{24}	a_{21}	a_{22}	a_{23}
a_{31}	a_{32}	a_{33}	a_{34}	a_{31}	a_{32}	a_{33}

Determinant =

$$a_{11}\left(a_{22}a_{33}a_{44} + a_{32}a_{43}a_{24} + a_{42}a_{23}a_{34} - a_{24}a_{33}a_{42} - a_{34}a_{43}a_{22} - a_{32}a_{23}a_{44}\right)$$
$$- a_{12}\left(a_{23}a_{34}a_{41} + a_{33}a_{44}a_{21} + a_{43}a_{24}a_{31} - a_{21}a_{34}a_{43} - a_{31}a_{44}a_{23} - a_{33}a_{24}a_{41}\right)$$
$$+ a_{13}\left(a_{24}a_{31}a_{42} + a_{34}a_{41}a_{22} + a_{44}a_{21}a_{32} - a_{22}a_{31}a_{44} - a_{32}a_{41}a_{24} - a_{34}a_{24}a_{42}\right)$$
$$- a_{14}\left(a_{24}a_{32}a_{43} + a_{31}a_{42}a_{23} + a_{41}a_{22}a_{33} - a_{23}a_{32}a_{41} - a_{33}a_{42}a_{21} - a_{31}a_{22}a_{43}\right)$$

This is the step by step to determine the determinant.

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a_{11}	a_{12}	a_{13}	a_{14}	a_{11}	a_{12}	a_{13}
a_{21}	a_{22}	a_{23}	a_{24}	a_{21}	a_{22}	a_{23}
a_{31}	a_{32}	a_{33}	a_{34}	a_{31}	a_{32}	a_{33}
a_{41}	a_{42}	a_{43}	a_{44}	a_{41}	a_{42}	a_{43}
a_{21}	a_{22}	a_{23}	a_{24}	a_{21}	a_{22}	a_{23}
a_{31}	a_{32}	a_{33}	a_{34}	a_{31}	a_{32}	a_{33}

$$a_{11}\left(a_{22}a_{33}a_{44} + a_{32}a_{43}a_{24} + a_{42}a_{23}a_{34} - a_{24}a_{33}a_{42} - a_{34}a_{43}a_{22} - a_{32}a_{23}a_{44}\right)$$

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a_{11}	a_{12}	a_{13}	a_{14}	a_{11}	a_{12}	a_{13}
a_{21}	a_{22}	a_{23}	a_{24}	a_{21}	a_{22}	a_{23}
a_{31}	a_{32}	a_{33}	a_{34}	a_{31}	a_{32}	a_{33}
a_{41}	a_{42}	a_{43}	a_{44}	a_{41}	a_{42}	a_{43}
a_{21}	a_{22}	a_{23}	a_{24}	a_{21}	a_{22}	a_{23}
a_{31}	a_{32}	a_{33}	a_{34}	a_{31}	a_{32}	a_{33}

$$- a_{12}\left(a_{23}a_{34}a_{41} + a_{33}a_{44}a_{21} + a_{43}a_{24}a_{31} - a_{21}a_{34}a_{43} - a_{31}a_{44}a_{23} - a_{33}a_{24}a_{41}\right)$$

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a_{11}	a_{12}	a_{13}	a_{14}	a_{11}	a_{12}	a_{13}
a_{21}	a_{22}	a_{23}	a_{24}	a_{21}	a_{22}	a_{23}
a_{31}	a_{32}	a_{33}	a_{34}	a_{31}	a_{32}	a_{33}
a_{41}	a_{42}	a_{43}	a_{44}	a_{41}	a_{42}	a_{43}
a_{21}	a_{22}	a_{23}	a_{24}	a_{21}	a_{22}	a_{23}
a_{31}	a_{32}	a_{33}	a_{34}	a_{31}	a_{32}	a_{33}

$$+ a_{13}\left(a_{24}a_{31}a_{42} + a_{34}a_{41}a_{22} + a_{44}a_{21}a_{32} - a_{22}a_{31}a_{44} - a_{32}a_{41}a_{24} - a_{34}a_{24}a_{42}\right)$$

$$
\begin{array}{cccc}
a_{11} & a_{12} & a_{13} & \mathbf{a_{14}} \\
a_{21} & a_{22} & a_{23} & a_{24} \\
a_{31} & a_{32} & a_{33} & a_{34} \\
a_{41} & a_{42} & a_{43} & a_{44} \\
a_{21} & a_{22} & a_{23} & a_{24} \\
a_{31} & a_{32} & a_{33} & a_{34}
\end{array}
\qquad
\begin{array}{ccc}
a_{11} & a_{12} & a_{13} \\
\mathbf{a_{21}} & \mathbf{a_{22}} & \mathbf{a_{23}} \\
\mathbf{a_{31}} & \mathbf{a_{32}} & \mathbf{a_{33}} \\
\mathbf{a_{41}} & \mathbf{a_{42}} & \mathbf{a_{43}} \\
\mathbf{a_{21}} & \mathbf{a_{22}} & \mathbf{a_{23}} \\
\mathbf{a_{31}} & \mathbf{a_{32}} & \mathbf{a_{33}}
\end{array}
$$

$$- a_{14}\left(\, a_{24}a_{32}a_{43} + a_{31}a_{42}a_{23} + a_{41}a_{22}a_{33} - a_{23}a_{32}a_{41} - a_{33}a_{42}a_{21} - a_{31}a_{22}a_{43}\right)$$