

# Spline-based signal reconstruction algorithm from multiple level crossing samples

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## Abstract

Commonly used analog-to-digital converters sample signals uniformly. The sampling rate is determined by maximal frequency in signal spectrum. In the general case the spectral content of the majority of signals encountered in our everyday life changes over time. The main idea behind level-crossing sampling is to take advantage of the characteristics of the input signal in order to minimize activity, power consumption and hardware complexity of the circuit that performs the digitizing [1].

For level-crossing sampling quantization levels are uniformly disposed along the amplitude range of the signal. A sample is captured only when the analog input signal crosses one of these levels. This means that in general samples are not uniformly spaced out in time and the sampling density depends on the signal's local properties. The amplitude of the captured sample is precise, but the time elapsed since the previous sample is measured by a local timer. The paper describes the process of reconstructing the signal waveform using its level-crossing samples.

Several signal reconstruction methods are developed for non-uniform sampling case [2], [3], [4]. The use of them typically limits the maximal length of the gaps between the sampling points. Simple iterative reconstruction algorithms are based on approximating methods that use families of functions called partitions of unity [5]. The idea is to interpolate the sampled bandlimited signal and filter it in order to remove high frequencies outside the origi-

nal signal bandwidth. Further the difference between the original sampling values and those of the previous approximation is used and an additive correction is generated, which leads to improved approximation. Using the partition of unity consisting of the triangular functions (well suited for level-crossing samples) leads to a piecewise linear interpolation of the signal. It is proved that if the maximal gap of the sampling set doesn't exceed the Nyquist criterion, then every bandlimited signal can be completely reconstructed from its sampling values iteratively using an ideal filter [5]. If the signal is sampled by level-crossings the maximal gap can exceed the Nyquist criterion depending on the signal's local properties and the number of quantization levels, making an exact reconstruction impossible. To overcome this obstacle, information about the signal's local sampling density, which is provided by level-crossings, is used.

As an example the signal with time varying spectral content is discussed. Performing a Fourier transform on such a signal multiplied by a window function, which is nonzero for only a short period of time, allows the determination of the local maximal frequency in the signal spectrum during this time interval. To estimate the global maximal frequency the length of the window function has to be equal to the duration time of the signal. Obviously the local maximal frequency can not exceed the global maximal frequency. Therefore iterative signal reconstruction in a local time interval is possible even if the maximal gap between the sampling points in this interval

exceeds the Nyquist criterion for the whole of the duration of the signal. To reconstruct the signal successfully the local maximal gap should not exceed the local Nyquist criterion, and a filter with corresponding bandwidth has to be used. The problem is to estimate the local maximal frequency using information about sampling moments in order to determine the local filter bandwidth.

For uniform sampling a reconstruction filter bandwidth can be determined from the signal sampling rate, because it corresponds to the maximal frequency in the signal spectrum. A similar approach can be used for estimating the local filter bandwidth. A defined number of successive samples is selected and the average sampling density for this time interval is calculated. In order to reduce the influence of big differences between the lengths of the gaps, they are sorted in ascending order and only the values from the middle part are used to estimate the average sampling density. Thereafter, a local filter bandwidth for a chosen time interval can be estimated on the basis of the sampling density obtained. Where the signal changes slowly, a small number of samples are captured and an average sampling density is lower making the filter bandwidth narrower. To improve the localization properties of complete reconstruction, the proposed algorithm uses the cardinal splines of the third order instead of a slowly decreasing sinc-function, which corresponds to ideal filtering.

The performance of the proposed algorithm is demonstrated on the basis of two different test-signals. The first one is a chirp with constant amplitude, while the second is a chirp with time varying amplitude. The results obtained by the iterative reconstruction using spline filters with adapting bandwidths show that the proposed method is quite successful. In addition to good accuracy, it offers fast reconstruction of the signal since the cardinal spline filter impulse response has to be calculated only once. To ensure even better results, it is worth developing more precise filter bandwidth estimation methods in the future. Speech signal processing can be cited as one of the potential application areas of the proposed algorithm. The level-crossing sampling technique reduces the number of samples and leads to effective

signal coding approaches.

#### REFERENCES

- [1] E. Allier, G. Sicard, L. Fesquet, M. Renaudin, "A New Class of Asynchronous A/D Converters Based on Time Quantization," Ninth IEEE International Symposium on Asynchronous Circuits and Systems, 2003.
- [2] K. Gröchening, H. Schwab, "Fast Local Reconstruction Methods for Nonuniform Sampling in Shift Invariant Spaces," SIAM Journal on Matrix Analysis and Applications, Vol. 24, No. 4, pp. 899-913, 2003.
- [3] A. Aldroubi, H. Feichtinger, "Exact Iterative Reconstruction Algorithm for Multivariate Irregularly Sampled Functions in Spline-Like Spaces: The  $L^p$ -Theory," Proceedings of the American Mathematical Society, Vol. 126, No. 9, pp. 2677-2686, 1998.
- [4] H.G. Feichtinger, K. Gröchening, "Theory and practice of irregular sampling," 1994.
- [5] T. Werther, "Reconstruction from irregular samples with improved locality", 1999.

## FURTHER READING

Click any one of the following links to be taken to a website which contains the following documents.

The following are some recent examples of Asynchronous ADC activity off the web.

[6 bit Asynchronous December 2006](#)  
[Asynchronous ADC In CAD Mentor Graphics](#)  
[Asynchronous Data Processing System](#)  
[ASYNCHRONOUS PARALLEL RESISTORLESS ADC](#)  
[Flash Asynchronous Analog-to-Digital Converter](#)  
[Novel Asynchronous ADC Architecture](#)  
[LEVEL BASED SAMPLING FOR ENERGY CONSERVATION IN LARGE NETWORKS](#)  
[A Level-Crossing Flash Asynchronous Analog-to-Digital Converter](#)  
[Weight functions for signal reconstruction based on level crossings](#)  
[Adaptive Rate Filtering Technique Based on the Level Crossing Sampling](#)  
[Adaptive Level-Crossing Sampling Based DSP Systems](#)  
[A 0.8 V Asynchronous ADC for Energy Constrained Sensing Applications](#)  
[Spline-based signal reconstruction algorithm from multiple level crossing samples](#)  
[A New Class of Asynchronous Analog-to-Digital Converters](#)  
[Effects of time quantization and noise in level crossing sampling stabilization](#)

Here is some more background information on Analog to Digital converters.

[A 1-GS/s 6-bit 6.7-mW ADC](#)  
[A Study of Folding and Interpolating ADC](#)  
[Folding ADCs Tutorials](#)  
[high speed ADC design](#)  
[Investigation of a Parallel Resistorless ADC](#)

Here are some patents on the subject.

[4,291,299 Analog to digital converter using timed](#)  
[4,352,999 Zero crossing comparators with threshold](#)  
[4,544,914 Asynchronously controllable successive approximation](#)  
[4,558,348 Digital video signal processing system using](#)  
[5,001,364 Threshold crossing detector](#)  
[5,315,284 Asynchronous digital threshold detector](#)  
[5,945,934 Tracking analog to digital converter](#)  
[6,020,840 Method and apparatus for representing waveform](#)  
[6,492,929 Analogue to digital converter and method](#)  
[6,501,412 Analog to digital converter including a quantizers](#)  
[6,667,707 Analog to digital converter with asynchronous ability](#)  
[6,720,901 Interpolation circuit having a conversio2](#)  
[6,850,180 SelfTimed ADC](#)  
[6,965,338 Cascade A D converter](#)  
[7,133,791 Two mean level crossing time interval](#)

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